Possibilistic information fusion using maximal coherent subsets

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Abstract— **When multiple sources provide information about the same unknown quantity, their fusion into a synthetic interpretable message is often a tedious problem, especially when sources are conflicting. In this paper, we propose to use possibility theory and the notion of maximal coherent subsets, often used in logic-based representations, to build a fuzzy belief structure that will be instrumental both for extracting useful information about various features of the information conveyed by the sources and for compressing this information into a unique possibility distribution.**

I. INTRODUCTION

When multiple sources deliver information tainted with uncertainty about some unknown quantity, aggregating this information can be a tedious task, especially when information is conflicting. This problem was first addressed in the framework of probability theory, and still constitutes an active area of research (see [1] for a recent review).

Some shortcomings of probabilistic methods are emphasized in [2], where it is shown that probabilistic methods tend to confuse randomness with imprecision. The shortcomings of the arithmetic mean (the most used and founded fusion operator for probabilities) are also discussed. Namely it tends to suggest, as being plausible, values none of the sources considered possible.

An alternative approach is to consider other theories of uncertainty, such as imprecise probabilities [3], evidence theory [4] or possibility theory [5]. These theories allow to faithfully model incomplete or imprecise data, a feature that probability theory arguably cannot account for. When it comes to aggregating data from multiple sources, these theories possess far more flexibility in the treatment of conflicting information, mainly due to the flexible use of set-operations (conjunction and disjunction).

In this paper, we will focus on uncertainty modeled by possibility distributions, for they can be easily elicited and interpreted as collection of confidence intervals, and are attractive from a computational viewpoint. On the other hand, possibility distributions can be judged not expressive enough in regard with available information (other theories should then be used).

Many fusion rules have been proposed to aggregate conflicting possibility distributions, using combinations of

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conjunction and disjunction operations, possibly exploiting external data (e.g. reliability of sources); see [6] for review. Most of these propositions result in one final possibility distribution built from the original ones provided by the sources. In this paper, we explore a fusion method based on maximal coherent subsets (an intuitive way of fusing information inspired from methods that handle inconsistency in logic [7]). However because we merge fuzzy information, the result will be a fuzzy belief structure.

After some preliminaries introduced in section II, section III explains how maximal coherent subsets are applied to obtain the fuzzy belief structure. section IV then presents some means to extract information from this structure. Finally, section V deals with the problem of getting one final possibility distribution.

II. PRELIMINARIES

A. Possibility theory

A possibility distribution $\pi(x)$ is a mapping from a space X to $[0, 1]$ and is formally equivalent to a fuzzy membership function μ s.t. $\mu(x) = \pi(x)$. One can interpret a possibility distribution on the real line as a set of nested confidence intervals [9]. From a possibility distribution, possibility and necessity measures are respectively defined as :

$$\Pi(A) = \sup_{x \in A} \pi(x)$$
$$N(A) = 1 - \Pi(A^c)$$

Where A^c stands for the complement of A. A possibility degree $\Pi(A)$ quantifies to what extent the event A is plausible, while the necessity degree quantifies the certainty of A. These measures can sometimes be interpreted as probability bounds

An α-cut E^α of the distribution π is defined as the set

$$E^\alpha = \{x | \pi(x) \geq \alpha\}$$

The core $c(\pi)$ and the support $s(\pi)$ of π respectively correspond to E^1 and $\lim_{\epsilon \to 0} E^\epsilon$

B. Fuzzy belief structure

A belief structure consists of a mapping m from subsets of a space X to $[0, 1]$ s.t. $\sum_{E \subseteq X} m(E) = 1, m(E) \geq 0$ and $m(\emptyset) = 0$. Sets E that have positive mass are called focal sets. From this mapping, we can again define two set-functions, the plausibility and belief functions, which read [4]:

$$Bel(A) = \sum_{E, E \subseteq A} m(E)$$

$$Pl(A) = \sum_{E, E \cap A} m(E) = 1 - Bel(A^c)$$

where the belief function quantifies the amount of information that surely supports A, and the plausibility reflects the amount of information that potentially supports A. When focal sets are nested, a belief structure is equivalent to a possibility distribution, and the belief (plausibility) function is also a necessity (possibility) measure. In this model the mass $m(E)$ should be interpreted as the probability of only knowing that the unknown quantity lies in E.

Zadeh [10] was the first to propose an extension of belief structures when focal sets are fuzzy sets. Here, we will note these fuzzy sets F_i. Since then, many proposals appeared that extend plausibility and belief functions when focal elements also are fuzzy (see, for example [11], [12]). In this paper, we retain Yen's [13] definition, which, in the discrete case, reads:

$$Pl_m(A) = \sum_{i=1}^{n} m(F_i) \sum_{\alpha_j} (\alpha_j - \alpha_{j-1}) \max_{w \in F_i^{\alpha_j}} \mu_A(w) \quad (1)$$

$$Bel_m(A) = \sum_{i=1}^{n} m(F_i) \sum_{\alpha_j} (\alpha_j - \alpha_{j-1}) \min_{w \in F_i^{\alpha_j}} \mu_A(w) \quad (2)$$

with $0 \leq \alpha_1 < \ldots < \alpha_j < \ldots \leq 1$ and where $F_i^{\alpha_j}$ is the α_j-cut of the fuzzy focal element F_i. The reason of choosing this generalization rather than another one is that the part involving fuzzy sets F_i in equations (1) and (2) is equivalent to computing the Choquet integral [14] of the (possibly fuzzy) event A over the possibility distribution $\pi_i = \mu_{F_i}$. We thus use linear operators in every part of the equation, which seems to us more coherent than using a mixing of linear operators and maximum/minimum based operators. Let us also notice that Yen's approach is a generalization of Smet's definition [15]. Although Yen's work is not based on these two considerations, but rather on optimization criteria, it is interesting to underly the fact that this generalization of belief structure to fuzzy focal sets has theoretical justifications. In fact, it comes down to reducing a random fuzzy set to a random set where each cut $F_i^{\alpha_j}$ has mass $m(F_i)(\alpha_j - \alpha_{j-1})$ [16].

C. Problem statement and illustration

In this paper, we will consider a set of n sources, each of them providing a possibility distribution π_i as their evaluation of an unknown quantity $x \in X$. We will then use maximal coherent subsets to summarize the information and then work on the resulting structure.

To illustrate our purpose, consider the following example : four sources (experts, computer code, sensor, . . .) all provide information in term of a best-estimate and a conservative interval, and the possibility distributions are supposed to have trapezoidal shapes. The information, represented in figure 1, is summarized in table I.

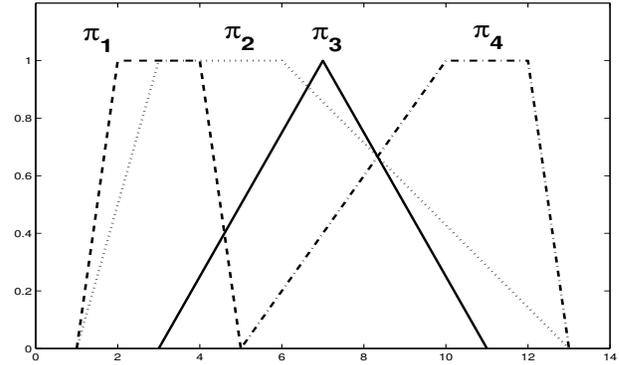| Source	Conservative interval	Best estimate
1	[1,5]	[2,4]
2	[1,13]	[3,6]
3	[3,11]	[7]
4	[5,13]	[10,12]

Fig. 1. Example graphical representation

III. MAXIMAL COHERENT SUBSETS METHOD

When no information is available about the sources reliability, and when these sources are conflicting, a reasonable fusion method should take account of the information provided by all sources (i.e. without discarding any). At the same time, it should try to gain a maximum of informativeness. Using the notion of maximal coherent subsets is a natural way to achieve these two goals. It consists of applying a conjunctive operator inside non-conflicting subsets of sources, and then to use a disjunctive operators between all these subsets [7]. We thus gain as much precision as possible while remaining in agreement with the information provided by every source. We now explain in detail how we apply this approach to possibility distributions on the real line.

A. Computing maximal coherent subsets of intervals

Let us first consider a set $N = \{1, \ldots, n\}$ of n intervals $I_i = [a_i, b_i]$. Using the method of maximal coherent subsets on these intervals comes down to finding every maximal subset $K_j \subset N$ s.t. $\cap_{i \in K_j} I_i \neq \emptyset$ and then to performing the union of these partial results (i.e. $\cup_j \cap_{i \in K_j} I_i$). An algorithm 1, that finds maximal coherent subsets, was given by Dubois et al. in [17]. Contrary to what happens in logic (where the exhaustive search for maximal coherent subsets of formulas is of exponential complexity), the algorithm 1 is linear in the number of intervals, and thus computationally efficient.

The algorithm is based on increasingly sorting the interval ends into a sequence $(c_i)_{i=1,\ldots,2n}$, and on the fact that each time (and only then) c_i is of type b (i.e. the end of an interval), followed by c_{i+1} of type a (i.e. the beginning of another interval), a maximally coherent intersection of intervals is obtained. Figure 2 illustrates the situation for

Algorithm 1: Maximal coherent subsets of intervals

> **Input**: n intervals
> **Output**: List of m maximal coherent subsets K_j
> List = $\emptyset$;
> j=1 ;
> $K = \emptyset$;
> Order in an increasing order
> $\{a_i, i = 1, \ldots, n\} \cup \{b_i, i = 1, \ldots, n\}$;
> Rename them $\{c_i, i = 1, \ldots, 2n\}$ with $type(i) = a$ if
> $c_i = a_k$ and $type(i) = b$ if $c_i = b_k$;
> **for** $i = 1, \ldots, 2n - 1$ **do**
>   **if** $type(i) = a$ **then**
>   Add Source k to K s.t. $c_i = a_k$;
>   **if** $type(i + 1) = b$ **then**
>   Add K to List ($K_j = K$) ;
> $j = j + 1$;
>   **else**
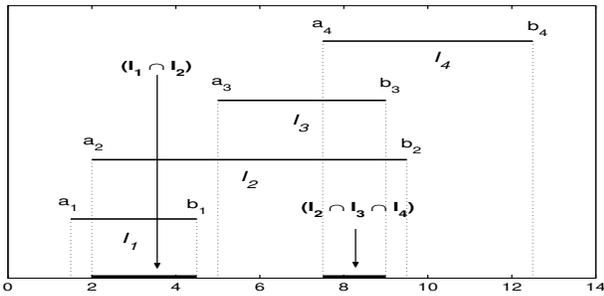>   Remove Source k from K s.t. $c_i = b_k$;

Fig. 2. Maximal coherent subsets on Intervals (0.5 α-cuts of example)

α-cuts of level 0.5 of our example. Using algorithm 1, we find two maximal coherent subsets : $K_1 = \{I_1, I_2\}$ and $K_2 = \{I_2, I_3, I_4\}$. After applying the maximal coherent subset method, the result is $(I_1 \cap I_2) \cup (I_2 \cap I_3 \cap I_4) = [2, 4.5] \cup [7.5, 9]$, as pictured in bold line on the figure.

B. Building the fuzzy belief structure

Let us come back to our n sources and their associated possibility distributions π_i supposed to be fuzzy intervals. At each level α, the α-cuts form a set of n intervals E_i^α. It is then possible to apply algorithm 1 to each of these sets of intervals : Let K_j^α be the subsets of intervals s.t. $\bigcap_{i \in K_j^\alpha} E_i^\alpha \neq \emptyset$. We then compute E^α the union of the intersection of subsets K_j^α as proposed in [8] :

$$E^\alpha = \bigcup_{j=1,\ldots,n(\alpha)} \bigcap_{i \in K_j^\alpha} E_i^\alpha \qquad (3)$$

where $n(\alpha)$ is the number of subsets K_j^α of maximal consistent intervals at a given level. In general, E^α is a union of disjoint intervals, and we don't have $E^\alpha \supset E^\beta \quad \forall \beta > \alpha$ (i.e. the result is not a possibility distribution, since α-cuts are not nested). In practice, for trapezoidal fuzzy intervals, there will be a finite set of values $0 = \beta_1 \leq \ldots \leq \beta_{k^*} \leq \beta_{k^*+1} = 1$ s.t.

the sets E^α will be nested for $\alpha \in (\beta_k, \beta_{k+1}]$. Algorithm 2 offers a simple method to compute threshold values β_k. It simply computes the heights of $\min(\pi_i, \pi_j)$ for every pair of possibility distributions π_i, π_j. Clearly, this value is the value of the α-cut after which π_i and π_j won't belong to the same coherent subset anymore.

Algorithm 2: Values β_k of fuzzy belief structure

> **Input**: n possibility distributions π_i
> **Output**: List of values β_k
> List = $\emptyset$;
> i=1 ;
> **for** $k = 1, \ldots, n$ **do**
>   **for** $l = k + 1, \ldots, n$ **do**
> $\beta_i = \max(\min(\pi_k, \pi_l))$;
>   i=i+1 ;
>   Add β_i to List ;
> Order List by increasing order ;

If we apply equation (3) for $\alpha \in (\beta_k, \beta_{k+1}]$, we end up with a non-normalized fuzzy set F_k with membership range $(\beta_k, \beta_{k+1}]$ (since sets E^α are nested in that range). We can then normalize it (so as to have the range $[0, 1]$) by changing $\mu_{F_k}(x)$ into

$$\frac{\max(\mu_{F_k}(x) - \beta_k, 0)}{\beta_{k+1} - \beta_k}$$

and assigning weight $m_i = \beta_{k+1} - \beta_k$ to this fuzzy set. By abuse of notation, we still denote F_k these normalized fuzzy focal sets in the sequel. Overall, we built a fuzzy belief structure $(\mathcal{F}, m)$ with weights m_i and associated normal focal sets F_k. The weights m_i can be interpreted as the confidence that would be given to the representation F_k by all the sources. Figure 3 gives an illustration of the result (before normalization), on the example. We see that, if we take the 0.5 α-cut, we find back the result of figure 2. The obtained fuzzy belief structure is thus a coherent "fuzzification" of the maximal coherent subset method used on classical intervals. Let us note that if all sources agree at least on a common value, the result is a single fuzzy focal set equivalent to $\pi(x) = \min_{i=1,\ldots,n} \pi_i(x)$ (usual conjunction). On the contrary, if every pair of sources is in a situation of total conflict (i.e. $\sup_{x \in X} \min(\pi_i, \pi_j) = 0 \quad \forall i \neq j$), then the result is a unique fuzzy focal set $\pi(x) = \max_{i=1,\ldots,n} \pi_i(x)$ (usual disjunction). Clearly, these two cases are extreme cases where one do not need complex mathematical tools to analyze the information.

Once the fuzzy belief structure $(\mathcal{F}, m)$ is computed, we can interpret it as a good representation of the information provided by the overall group of sources. Nevertheless, it can be hard to draw conclusions or useful information directly from it (see figure 3 to be convinced). In the following section, we present various indices that can be useful to the analyst.
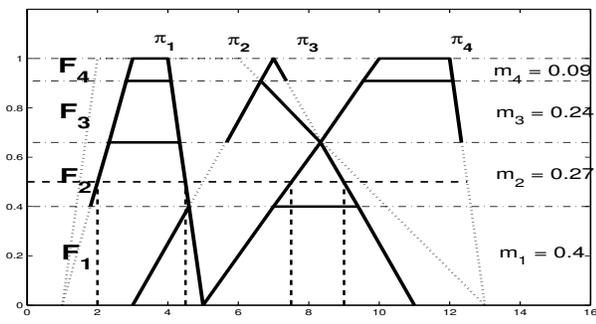
Fig. 3. Result of maximal coherent subset method on example (—) and 0.5 α-cut (- - -)

IV. EXTRACTING INFORMATION

In this section, we present some indices or information that can help in the analysis of the results obtained in the previous section.

A. Finding groups of coherent sources

For each value in interval $(\beta_k, \beta_{k+1}]$, applying algorithm 1 will give the same maximal coherent subsets $K_j^{(\beta_k, \beta_{k+1}]}$ of sources. We will thus have a finite collection of such subsets. Moreover, by increasing the value of β_k, we go from the largest sets of agreeing sources (i.e. those for which the supports of distributions π_i intersect), to the smallest sets of agreeing sources (i.e. those for which cores intersect). Subsets $K_j^{(\beta_k, \beta_{k+1}]}$ can be interpreted as the clusters of sources that agree up to a confidence level β_{k+1}.

Analyzing these clusters can give some information as to which groups of sources are consistent, i.e. agree together with a high confidence level (possibly using some common evidence to supply information) and which ones are strongly conflicting with each other (and which items of information are plausibly based on different pieces of evidence). The groups in our example are summarized in the following table

Subsets	Clusters	Max. Conf. level
$K^{(0,0.4]}$	$[1,2,3][2,3,4]$	0.4
$K^{(0.4,0.66]}$	$[1,2][2,3,4]$	0.66
$K^{(0.66,0.91]}$	$[1,2][2,3][4]$	0.91
$K^{(0.91,1]}$	$[1,2][3][4]$	1.0

In our example, only few conclusions can be drawn from the clusters, showing that, if this kind of summary can be useful, it is not sufficient. Results show that some sources are totally conflicting (since there is more than one subgroup for in $K^{(0,\beta_1]}$), and that source 4 looks more isolated than the three others (at a confidence level higher than 0.66, it is strongly conflicting with all other sources). This type of analysis can trigger further investigations on reasons for conflict.

B. Measuring the gain in precision

We then propose an index that measures how much precision is gained by applying maximal coherent subsets method to a set of n possibility distributions. Let $\pi_\cup$ be the disjunction s.t. $\pi_\cup = \max_{i=1,...,n} \pi_i$. We consider that the overall imprecision of the information provided by all the sources is equal to

$$IP = |\pi_\cup| = \int_X \pi_\cup(x)dx$$

where $|\pi_\cup|$ is the fuzzy cardinality of $\pi_\cup$, an extension of the cardinality of an interval (the cardinality being a natural candidate to measure imprecision). Once we have applied the maximal coherent subset method, the imprecision of the fuzzy belief function Bel_i can be measured as

$$IP' = \sum m_k |F_k|$$

$(IP - IP')/IP$ is a normalized index that quantify the precision gained between the two representations. This index is equal to 0 in case of total conflict and converges to 1 as sources tend to fully agree on a single value (i.e. $\exists! \, x$ s.t. $\pi_i(x) = 1 \quad \forall i$).

In our example, we have $IP = 11.195$, $IP' = 5.412$ and the normalized index is 0.52, which indicates a reasonable gain of precision after application of maximal coherent subsets method. Note that IP'/IP can also be interpreted as a measure of global conflict between sources, since it is maximum in case of total conflict and minimum in case of total agreement on a single value.

C. Group confidence in an event, in a source

Since we consider the fuzzy belief structure $(\mathcal{F}, m)$ resulting from the maximal coherent subset method as a good representative of the group of sources, plausibility and belief functions of an event A can be interpreted respectively as an upper and a lower confidence level given to A by the sources. In particular, if $A = \pi_i$, plausibility and belief can be interpreted as upper and lower "trust" in the information given by source i in view of all the sources.

In our example, values $[Bel_m(\pi_i), Pl_m(\pi_i)]$ for sources 2 and 4 are, respectively, $[0.38, 1]$ and $[0, 0.93]$ (using equations (2) and (1)). We see that information provided by the second source is judged totally plausible by the group, and also strongly supported (indeed, source 2 is undoubtedly the less conflicting of the four). Because one source completely disagrees with source 4, its belief value drops to zero, but the information delivered by it is nevertheless judged fairly plausible (since source 4 is not very conflicting with sources 2 and 3).

Belief and plausibility functions are natural candidates to measure the overall confidence in a source, but their informativeness can sometimes be judged too poor. Indeed, if a distribution π_i given by a source i is in total conflict with the others, the resulting fuzzy belief structure $(\mathcal{F}, m)$ will give the following measures for $\pi_i : [Bel_m(\pi_i), Pl_m(\pi_i)] = [0, 1]$, a result that can't be less informative. An alternative would be to take the fuzzy equivalent of the so-called pignistic probability, namely

$$BetP(A) = \sum m(F_k) \frac{|F_k \cap A|}{|F_k|} \qquad (4)$$

where $|F_k \cap A|/|F_k|$ is the degree of subsethood of F_k in A. This pignistic probability is zero if A is strongly conflicting with every focal set F_k and one if every F_k is included in A (here, F_k is included in A iff $\mu_{F_k}(x) < \mu_A(x) \forall x$). In the example, equation (4) applied to sources 2 and 4 respectively gives confidence 0.80 and 0.49, confirming that source 2 is more trusted by the group than source 4.

D. Measuring confusion

There are many proposals to extend Shannon's or Hartley's like measures to belief functions (for a recent review, see [18]). Since most of them only use focal sets masses, plausibility and belief functions, they can be straightforwardly extended to fuzzy belief structures (for measures that use entropies on families of probabilities, extension is more problematic). However, an interpretation of these measures in term of source behavior is not always easy to give. In this paper, we will restrict to a measure extensively studied in [19], [20], for which we are able to give a meaningful interpretation in term of sources.

The measure is called the measure of confusion, and reads

$$C(\mathcal{F}, m) = \sum_{F_k} m(F_k) Bel(F_k) \tag{5}$$

for fuzzy belief structures. For usual non-fuzzy belief structures, $C(\mathcal{F}, m)$ reaches its maximum (1) when $(\mathcal{F}, m)$ has only one focal set that is a crisp interval, and its minimum (0) if weights are uniformly distributed amongst a maximal number of focal elements forming an antichain.

In our framework and in term of possibility distributions given by sources, the maximum of $C(\mathcal{F}, m)$ is reached if and only if each π_i is a crisp interval (i.e. sources give precise bounds). As for the minimum, $C(\mathcal{F}, m)$ is all the smaller as :

1) the resulting fuzzy belief structure has more fuzzy focal sets F_k
2) Weights are more uniformly distributed among elements F_k
3) There are more pairs of focal sets F_k, F_j $j \neq i$ there is a value x s.t. $x \in c(F_j) \rightarrow x \notin s(F_k)$

where $c(F_k), s(F_k)$ are respectively the core and support of the focal set F_k. The last constraint insures that the Choquet integral will be zero in equation (2), except for $F_k = F_j$, which imply that $Bel(F_k)$ will be minimal. This index is a good way to measure confusion between sources, since it increases as sources become fuzzy and as conflict levels are uniformly distributed amongst pairs of sources.

Given the n sources supplying trapezoidal fuzzy intervals, the maximal number of fuzzy focal sets resulting from our method is $\frac{n(n-1)}{2} + 1$. Moreover, we have that $Bel(F_k) \geq 0.5m(F_k)$, due to the fuzziness of sets F_k. Hence, we have that $\inf(C(\mathcal{F}, m)) = 0.5(\frac{n(n-1)}{2} + 1)^{-2}$, a value that may quickly vanish, even for a small number of sources (0.03 for 3 sources and 0.004 for 5 sources). In our example, $C(\mathcal{F}, m) = 0.164$, suggesting a fairly confusing situation. This result indicates that the information is very scattered,
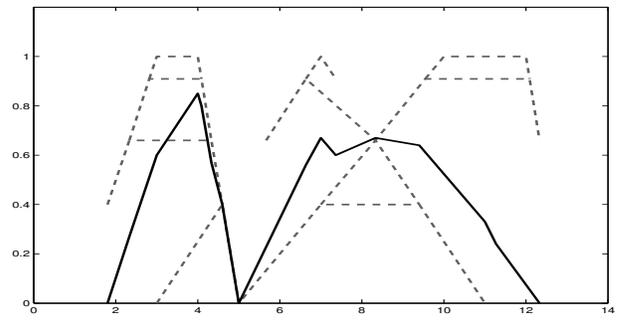

Fig. 4. Contour function π_c (—) with fuzzy focal sets (- - -)

and that some of the sources are perhaps considering different issues (sensors focusing on different objects, some experts having misunderstood the questions, ...). Let us notice that this index of confusion has a different meaning from the index of global conflict introduced above. In the case when every source provides pairwise disjoint intervals, the index of confusion will be minimal, while the global conflict will be maximal.

In our case, the fact that the measure of confusion mentioned above is not subadditive (an often heard objection), nor simply additive, is not very annoying here, since we do not need to decompose fuzzy belief structures nor to build Cartesian products of them.

V. Building a final distribution

As seen in previous sections, the maximal coherent subset method allows to extract useful information about sources and their assessment of an unknown quantity. Nevertheless, it can be hard to directly use the fuzzy belief structure representation in practical problems (such as uncertainty propagation through a mathematical model). In this case, a method that derives a unique distribution from a fuzzy belief structure $(\mathcal{F}, m)$ is needed.

A natural candidate is to build the contour function of the fuzzy belief structure

$$\pi_c(x) = Pl(x), \quad \forall x,$$

which boils down to computing the weighted arithmetic mean of the membership functions of (normalized) fuzzy focal sets F_k, the weight of F_k being equal to m_k. One can then normalize the resulting distribution π_c and/or take its convex hull if needed.

Figure 4 shows the contour function π_c with the fuzzy focal sets in the background. figure 5 shows the same function, once it is normalized and convexified, together with the original distributions in the background.

The final result is a bimodal distribution, with one mode centered around value 8 and the other with a value of 4, this last value being the most plausible. This is is so because these areas are the only ones supported by three sources that more or less agree together. We can expect that the true value lies in one of these two areas, but it is hard to tell which one. Indeed, in this case, one should either take the normalized
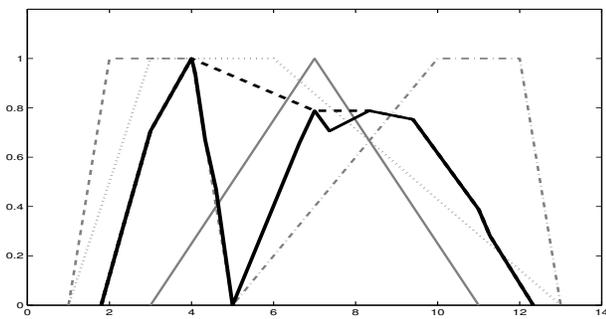
Fig. 5. Contour function, normalized (—) and convexified (- - -), with original distributions (gray lines)

convex hull of π_c as the final representation of the parameter X, or find out the reason for the conflict (if feasible).

VI. CONCLUSIONS

We have outlined a method that use the notion of maximal coherent subsets in the case where many sources express their uncertainty in term of possibility distributions. This method is simple (it can be applied without any additional information, and only use linear operations) and the way it summarizes information is intuitively attractive (maximal coherent subsets are implicitly used in our every day decisions).

We have proposed various ways to get useful information from the result of this method. More specifically, we concentrated on how to characterize the situation in term of sources (which sources agree/disagree and to which level, how to measure overall confusion, information gain or overall confidence in one source or in a particular event). This kind of information is useful to figure out where future efforts should be spent (to find the causes of a conflict, or why a set of sources provide the same information, …).

We have also proposed some way to get a final distribution coherent with the available information, using the fuzzy belief structure resulting from our method. This allow the decision maker to build a synthetic distribution, easy to understand and to manipulate, which is a good representative of the information delivered by the sources.

The method proposed here is intended to be generic, and does not depend on the nature of sources. It uses a natural logical approach to the fusion of inconsistent information within a sound and efficient numerical framework. The IRSN is planning to use this kind of methods to analyze information resulting from BEMUSE [21], an international benchmark concerning uncertainty analysis of thermal-hydraulic codes in nuclear safety.

As mentioned above, the maximal coherent subset method assumes that no specific information concerning the sources are available, and is applicable in a symbolic framework as well as on metric spaces (e.g. the real line). Of course, there are cases where such information are available, or where the metric should be taken into account. Thus, further work will be to integrate in a meaningful way such information into the fusion process, while keeping the idea of using maximal coherent subsets.

Let us notice that, instead of considering the set of fuzzy focal elements generated by our method, one could consider a continuous belief function [22] with a uniform density of weights distributed over α-cuts F_k^α. An interesting work would be to compare the method presented here with calculi made with this density.It is also necessary to compare the results of this method with other ones designed to aggregate conflicting possibility distributions (such as the ones described in [23], [24]).

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